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QCD AND HIGH ENERGY HADRONIC INTERACTIONS

On the role of the strange quark and its mass in the chiral phase transition

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The evolution of the chiral condensate with the temperature is studied using SU(3) Chiral Perturbation Theory and the virial expansion. We observe a decrease of the melting temperature of the non-strange condensate compared with the SU(2) case. Due to the larger mass of the strange quark we also find an slower temperature evolution of the strange condensate compared with the non-strange condensate.

There is a growing interest in the QCD phase diagram, in the transition from a hadron gas to a quark gluon plasma and in the existence of a chiral phase transition. In this note we review a rather simple approach [1] to describe the evolution of the chiral condensate, namely, the virial expansion [2] of a dilute gas made of interacting pions, kaons and etas. Up to second order, for most thermodynamic properties it is enough to know the low energy scattering phase shifts of the particles, which could be taken from experiment. However, in order to study the quark condensates, i.e. derivatives of the pressure with respect to the quark masses, a theoretical description of the mass dependence of the scattering amplitudes is needed. For that purpose we turn to Chiral Perturbation Theory (ChPT) [3], which provides a remarkable description of low energy hadronic interactions. Within ChPT the pions, kaons and the eta are identified as the Goldstone Bosons of the QCD spontaneous chiral symmetry breaking (pseudo Goldstone bosons, due to the small quark masses). The ChPT Lagrangian is the most general derivative and mass expansion respecting the symmetry constraints built out of $\pi's$, K's and the η. At one loop any calculation can be renormalized in terms of a finite set of parameters, L_k, H_k.

When SU(2) ChPT is applied to a gas where only the pions have interactions [4], the phase transition critical temperature can be estimated to be $T_c \simeq 190\,\text{MeV}$. It has also been shown that the perturbative calculation is analogous to the second order virial expansion if the interacting part of the second virial coefficient is obtained from the one loop $\pi\pi$ scattering lengths.

Recently[1] we have extended this approach to the SU(3) case, in order to study the effect of the strange quark and its large mass. Indeed, the chiral phase transition can be rather different for the SU(2) and SU(3) cases, and a larger condensate thermal suppression is expected with an increasing number of light flavors [5], since, intuitively, the existence of more states favors entropy and disorder at the expense of the ordered (condensed) phase. It is estimated that this so-called "paramagnetic" effect lowers the T_c down to 150 MeV [6] (in the chiral limit). Within our approach we will check this results from the hadronic phase with the physical values of the masses. In addition, within SU(3) we can study the $\langle\bar{s}s\rangle$ condensate. In particular, let us recall that quark masses play the same role as magnetic fields in ferromagnets. As we need a higher temperature to disorder a ferromagnet when there is a magnetic field along the direction of the magnetization, so it has been found that the SU(2) chiral condensate melts at a higher temperature when taking masses into account[4]. Since the strange quark mass $m_s > m_u, m_d$, we expect this "ferromagnetic" effect to raise the critical $\langle\bar{s}s\rangle$ temperature compared to that of $\langle\bar{q}q\rangle$.

For a gas made of three species: $i = \pi, K, \eta$, the pressure relativistic virial expansion [7,2] is :

$$\beta P = \sum_i B_i(T)\xi_i + \sum_i \left(B_{ii}\xi_i^2 + \frac{1}{2}\sum_{j\neq i} B_{ij}\xi_i\xi_j \right) ..., \tag{1}$$

where $\beta = 1/T$ and $\xi_i = \exp(-\beta m_i)$. Expanding up to the second order in ξ_i means that we only consider binary interactions. For a free boson gas, the first and second coefficients are

$$B_i^{(0)} = \frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 e^{-\beta(E(p)-m_i)}, B_{ii}^{(0)} = \frac{g_i}{4\pi^2}\int_0^\infty dp\, p^2 e^{-2\beta(E(p)-m_i)}, B_{ij}^{(0))} = 0 \text{ for } i \neq j, \tag{2}$$

where the degeneracy is $g_i = 3, 4, 1$ for π, K, η, respectively. The interactions enter through [7,4]:

$$B_{ij}^{(int)} = \frac{\xi_i^{-1}\xi_j^{-1}}{2\pi^3}\int_{m_i+m_j}^\infty dE\, E^2 K_1(E/T) \sum_{I,J,S}(2I+1)(2J+1)\delta_{I,J,S}^{ij}(E), \tag{3}$$

where K_1 is the first modified Bessel function and the $\delta_{I,J,S}^{ij}$ are the $ij \to ij$ elastic scattering phase shifts of a state ij with quantum numbers I, J, S (J being the total angular momentum and S the strangeness). Since we are interested in $T < 250\,\text{MeV}$, it is enough to consider $ij = \pi\pi, \pi K$ and $\pi\eta$ in the second virial coefficients.

Quark masses enter the free energy density z, as $m_q\bar{q}q$. Since in z only the pressure depends on the temperature, i.e., $P = \epsilon_0 - z$ (ϵ_0 being the vacuum energy density) we have:

$$\langle\bar{q}_\alpha q_\alpha\rangle = \frac{\partial z}{\partial m_{q_\alpha}} = \langle 0|\bar{q}_\alpha q_\alpha|0\rangle - \frac{\partial P}{\partial m_{q_\alpha}}. \tag{4}$$

where $q_\alpha = u, d, s$. Note that at $T = 0$ we recover $\langle\bar{q}_\alpha q_\alpha\rangle = \langle 0|\bar{q}_\alpha q_\alpha|0\rangle = \partial\epsilon_0/\partial m_{q_\alpha}$. Let us emphasize that for the chiral condensate *we need the dependence* of $\delta(E)$ *on the quark masses* as well as a value for the vacuum expectation value. For that information we turn to ChPT, that deals with $\pi's$, K's and η, and thus we translate eq.(4), in terms of meson masses:

$$\langle\bar{q}_\alpha q_\alpha\rangle = \langle 0|\bar{q}_\alpha q_\alpha|0\rangle \left(1 + \sum_i \frac{c_i^{\bar{q}_\alpha q_\alpha}}{2m_i F^2}\frac{\partial P}{\partial m_i}\right), \quad \text{with} \quad c_i^{\bar{q}_\alpha q_\alpha} = -F^2 \frac{\partial m_i^2}{\partial m_{q_\alpha}} \langle 0|\bar{q}_\alpha q_\alpha|0\rangle^{-1}. \tag{5}$$

where, as before, $i = \pi, K, \eta$. The c formulae to one loop can be easily obtained from those of $\langle 0|\bar{q}q|0\rangle$, $\langle 0|\bar{s}s|0\rangle$ and the one loop dependence of the meson masses on the quark masses, given a long time ago in [3]. The only relevant comment is that the c depend on the chiral parameters L_k , for $k = 4...8$, and H_2. There are several L_k and H_2 determinations from meson data and it makes little difference for our conclusions which one to use. For further details see ref.[1].

Finally, we want $\partial P/\partial m_i$, and for that we need the theoretical description of the meson-meson scattering phase shifts. The meson-meson amplitudes to one-loop in SU(3) ChPT have been given in [3,8,9]. These expressions have to be projected in partial waves of definite isospin I and angular momentum J. The complex phases of these partial waves are the $\delta_{I,J}(E)$ in eq.(3).

The results using the one-loop ChPT amplitudes, both in SU(3) and SU(2), can be seen in Fig.1 [1] . The virial coefficients and $\partial P/\partial m_i$ have been calculated numerically. Note that in the isospin limit the u and d condensates are equal, and we have used the notation $\langle 0|\bar{q}q|0\rangle \equiv \langle 0|\bar{u}u + \bar{d}d|0\rangle$. For the figures, we have represented the chiral condensate over its vacuum expectation value, $\langle \bar{q}_\alpha q_\alpha\rangle/\langle 0|\bar{q}_\alpha q_\alpha|0\rangle$, so that all of them are normalized to 1 at $T = 0$. For further details of the calculation (parameters, etc... see [1]). The melting temperature estimates are: $T_m^{\langle \bar{q}q\rangle, SU(2)} = 231^{+30}_{-10}$ MeV, $T_m^{\langle \bar{q}q\rangle, SU(3)} = 211^{+19}_{-7}$ MeV and $T_m^{\langle \bar{s}s\rangle} = 291^{+37}_{-35}$. Note that all these temperatures are strongly correlated, i.e. the larger in SU(2), the larger in SU(3), in particular, we find: $T_m^{\langle \bar{q}q\rangle, SU(2)} - T_m^{\langle \bar{q}q\rangle, SU(3)} = 21^{+14}_{-7}$ MeV. This in good agreement with the lattice results [6] in the chiral limit. About 5 of these MeV are due to free kaons or etas[4]. However the rest of the effect is due to the different SU(3) and SU(2) meson mass one-loop dependence on the quark masses. In Figure 1 we have separated the size of the different contributions. Furthermore, we can see that $\langle \bar{s}s\rangle$ melts slower than $\langle \bar{q}q\rangle$. In particular, $T_m^{\langle \bar{s}s\rangle} - T_m^{\langle \bar{q}q\rangle} = 80^{+25}_{-40}$ MeV. We have also estimated the effect of adding other, more massive, free hadrons and their contribution decreases $T_m^{\langle \bar{q}q\rangle}$ by 7-12 MeV, and $T_m^{\langle \bar{s}s\rangle}$ by 15-23 MeV.

Of course, with the second order virial approach we cannot generate a singularity in T like that associated to a phase transition. Indeed, our curve does enter into the negative region, whereas, in reality, and due to the small explicit symmetry breaking caused by the quark masses, the condensate should only vanish in the $T \to \infty$ limit. Still, we give the complete melting temperature as a reference to ease the comparison between different curves and previous works.

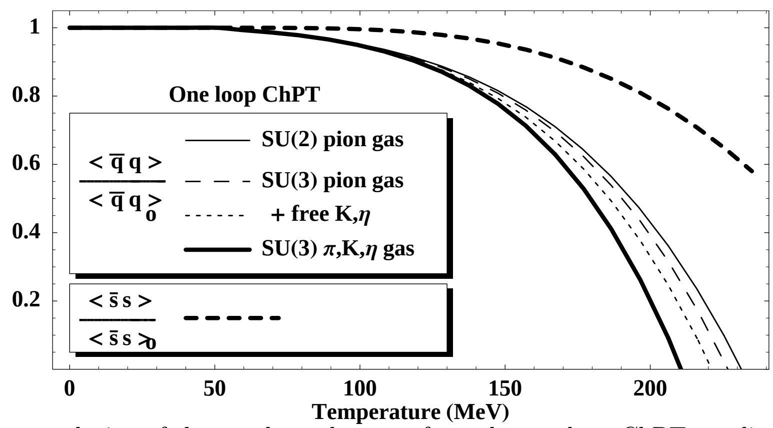

Figure 1: Temperature evolution of the quark condensates from the one loop ChPT amplitudes (extrapolated to zero for reference).

Let us recall that ChPT is an effective approach that is only valid at low temperatures so that beyond 100 or 150 MeV the curves in Fig.1 should be considered as a plausible extrapolation. However, note that both the ferromagnetic and paramagnetic effects are already visible at low temperatures. It is possible, however, to extend ChPT up to $E \simeq 1.2\,\mathrm{GeV}$ by means of unitarization methods [10,9]. These techniques resum the ChPT series respecting unitarity but also the low energy expansion, *including the mass terms.* In particular, it has been shown that the coupled channel Inverse Amplitude Method (IAM) provides an accurate description of the complete meson-meson interactions below 1.2 GeV, generating dynamically six resonances (and their associated poles): σ, f_0, ρ, a_0, κ, K^*, from the one-loop ChPT expansion. This approach can be extended systematically to higher orders (see the two last references in [10]). To estimate the effect of higher energies, we will use the IAM fit[9] to meson-meson data, which was also shown to give very accurate scattering lengths (only dependent on the masses) with a set of fitted L_k compatible with previous determinations.

In Figure II we show the results using the IAM phase shifts. The continuous line corresponds to the central values, and the shaded areas cover the one standard deviation uncertainty due only to the statistical errors in the parameters from a MINUIT IAM fit. These areas have been obtained from a Montecarlo gaussian sampling of L_k and H_2 within their errors. To obtain a conservative estimate of the error we have also sampled a conservative range provided in [9] which estimates some systematic uncertainties but that does not respect any correlation between chiral parameters. In such case, the uncertainty is covered by the area between the dotted lines. The SU(3) IAM estimate of the melting temperatures are $T_c^{\langle\bar{q}q\rangle,SU(3)} = 204^{+3}_{-1}\,(^{13}_{5})\,\mathrm{MeV}$ and $T_c^{\langle\bar{s}s\rangle} = 304^{+39}_{-25}\,(^{120}_{65})\,\mathrm{MeV}$, where the errors in parenthesis are the conservative range. Again, all temperatures are strongly correlated and we find $T_c^{\langle\bar{q}q\rangle,SU(2)} - T_c^{\langle\bar{q}q\rangle,SU(3)} = 31.50^{+1.29}_{-0.03}\,(^{9}_{8})\,\mathrm{MeV}$. The effect of the higher energies seems to be an slight decrease of all temperatures by about 5 MeV. Once more we find a slower evolution of the strange condensate due to the strange quark larger mass, in particular, we find $\Delta T_c = T_c^{\langle\bar{s}s\rangle} - T_c^{\langle\bar{q}q\rangle} = 100^{+36}_{-29}\,(^{120}_{80})\,\mathrm{MeV}$.

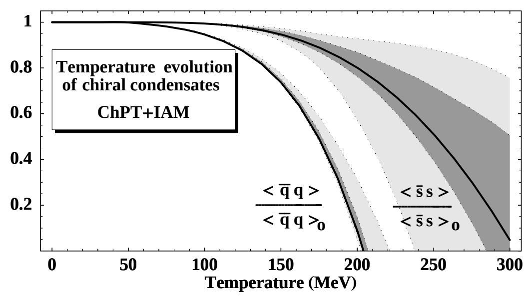

Figure 2: Quark condensate temperature evolution from IAM ChPT amplitudes. The shaded areas cover the uncertainties in the chiral parameters. (Curves extrapolated to zero only for reference).

To summarize, we have studied the SU(2) and SU(3) temperature evolution of the chiral condensates *in the hadronic phase*. The thermodynamics of the hadron gas has been obtained from the virial expansion and Chiral Perturbation Theory. Our results clearly show a significant decrease, about 20 MeV, of the non-strange condensate critical temperature, from the SU(2) to the SU(3) case. In addition, they suggest an slower melting of the strange condensate, shifted by 70-80 MeV with respect to the non-strange condensate, due to the different quark masses.

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